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BRITTLE SOLID UNDER COMPRESSION. PART I: GRADIENT MECHANISMS OF MICROCRACKING

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Abstract—The venerable problem of the origins of cracking and failure of a brittle heterogeneous solid (heterogen) under compression is analyzed here from a new point of view, with the cause of its microcracking and atrophy (degeneration) under load, found, first of all, in the differences in Poisson's ratio of its components.

Four fundamental phenomena, found in experiments, are the basis of the new approach to the behavior and failure of a *heterogen* under compression. (1) The non-linear part of the ascending branch of SSc is due to formation and accumulation of *stable* microcracks. (2) The intrinsic elastic modulus *remains constant* up to the peak point. (3) Concrete and rock fail *in splitting*. (4) The pattern of SSc *is the same* for different types of concrete and rock under different types of loading.

It is found that some *kinds of gradient mechanisms* can induce *local transverse strains of tension* and cause microcracking in a heterogen *under compression*. The *first* creates local strain gradients among the components due to difference in their Poisson extension, when the components with lower Poisson's ratio are tensioned in the lateral direction. There is also a mechanism which creates an internal thrust due to *gradient in the elastic moduli* of the component.

It is shown that, in a brittle solid built from randomly oriented crystals, a population of *laterally tensioned crystals*, called "acrons", are created due to gradients in Poisson's ratio of a single crystal along its three axes. The models of gradient strain in the acrons are given, including the equation of critical strains. The problem of crystals acting as "pistons" due to a process of sliding is also discussed.

The gradient models explain the appearance of microcracks and their stochasticity and why, instead of growing into *macrocracks*, they are stable, in good accordance with a vast number of experiments. Gradient mechanisms, especially that of Poisson, are *universal, descriptive and based on measurable parameters*. They suffice to exhaust the bearing capacity of a heterogen under increasing compression without recourse to shear stresses. They affect every heterogen under compression: rock materials, concrete, ceramics—and do not need initial microcracking to initiate and realize the process of atrophy (degradation) of the brittle solid. © 1997 Elsevier Science Ltd.

DEFINITIONS

At first a brittle heterogeneous solid will be referred to as a *heterogen*. A *matrogen* is a special important class of heterogens, which consist of a continuous *matrix* and of *particles* (aggregate) that "float" in the matrix without mutual contact. In an artificial matrogen, like concrete, the granulometry is given and kept under very close control. A heterogen built of contacted particles or grains can be called a *patrogen*. There can be a 'glue' between the particles, but it does not create a matrix. The case of interest is a *crystalon*, a kind of patrogen which is built of *randomly oriented crystals* almost without glue components.

NOMENCLATURE

Heterogen	brittle heterogeneous solid
SSc	the curve of stress-strain relationship
ε	longitudinal strain
ε_2	lateral strain induced by load
ε, ε^*	transverse strains and their gradients, respectively
ε^-	lateral strain induced by Poisson extension
σ, σ_2	longitudinal and lateral stress, respectively
E	elastic modulus
ν	Poisson's ratio
δ	gradient factor
a, m	indices of the components

1. RUPTURE OF BRITTLE SOLID

1.1. *State of the art*

In traditional description, the limiting strength of brittle solids is considered with the failure-inducing peak load, whereby the failure concept is associated with development of a major crack. Today this definition is obsolete. With more precise specimen-testing techniques, the descending branch in the *stress-strain curve* (SSc) is well recognized, and an ever-growing volume of findings regarding its features and design applications is being published. The upshot is that, in spite of the inherent brittleness of concrete and rock the maximum stress is not the endpoint of the loading curve; although it still represents the peak response of the specimen, it is no longer associated with total failure. In other words, it can no longer be identified with the moment of rapid collapse of the material following the onset and development of a major crack. Hence the need has arisen to bring out the "cushioned, soft" effects whereby the resistance of the material reduces gradually as the stress level increases. Thus the strength problem is a part of a general one—that of describing the state changes in the material which accompany the increase in the strains and stresses.

This problem is under intensive study in a number of directions, which may be classified according to approach and to the type of the considered physical process, namely:

Models of macrofracture (fracture mechanics);
Meso-models (damage analysis);
Local (micro)mechanical models of constituent interactions;
Models on the molecular level;
Abstract models (purely mathematical treatment).

In *fracture mechanics*, Mindess (1983), failure is attributed to an initially present major crack. Today it is clear that this scheme (referred to as "Mode I") cannot by itself account for the physical aspects of damage and strength of brittle solids.

To quote NMAB Report (1983), "... The committee identified two separate behavior patterns in compressive fracture. The first is an *extrinsic* behavior pattern resulting from *large cracks*, comparable with the size of the part, amenable to normal approaches of fracture mechanics...

The second is an *intrinsic* behavior pattern, resulting from the accumulation of dormant microcracks and culminating in terminal shear faulting that has been analyzed previously as a constitutive instability, *not amenable to treatment by normal procedures of fracture mechanics*".

Naturally, in fracture mechanics, priority is given to the domain from the point of peak stress down the descending branch of the SSc. The fracture approach is applied in a considerable number of works on fracture energy determination—as a basic characteristic for prediction of concrete failure based on measurements of the fracture energy [Swamy (1971), Shah (1985), van Mier (1986), Malvar (1987), Bazant (1987)]. The difficulties inherent in the fracture approach are properly illustrated in Mindess (1983).

The *molecular approach* is mostly concerned with the structure of hard cement paste, Feldman (1968), and existing data are qualitative rather than quantitative.

As regards the *mathematical approach*, considerable effort has been invested in searches for abstract stress-strain relationships (of which more than twenty have been proposed to date), without reference to the mechanism of the microcracking process (Popovics, 1971). With this lack of physical meaning, none of them can claim superiority.

The (relatively recent) so-called *damage model* approach deals with the nonlinear part of the *ascending branch*—in terms of the microcracking effect (Kzajcionovic, 1986; Lemaitre, 1987).

Here the material is considered at the level of "unit cells" containing a statistically valid sample of weak spots and microcracks which exists before the loading and is treated by *micromechanical models*. Yet it was established experimentally long ago, that the main phenomenon of this stage is accumulation of stable microcracks, which begin to merge near the strength point, forming macrocracks (with consequent division of the solid in parts), (Berg (1950, 1961, 1971), Slate (1963, 1981a, 1981b, 1986), Li and Nordlund (1993)).

A number of difficulties can be associated with the damage models:

a. What are the cause and the rules of the *onset of microcracking*? Where does the tension in a fully compressed solid come from?

b. Why does the tensile effect invariably "pick" *new* sound spots in concrete and rock, creating *new* microcracks with increasing stress—instead of opening up existing ones? Or, why do microcracks, once induced, remain dormant?

c. Why are the planes of new microcracks, induced under increasing load, oriented longitudinally, in the direction of compression?

Possible answers to these questions are given below in terms of transverse gradient strains.

1.2. *Griffith's point of view*

In view of Griffith's role in fracture mechanics, it may be of interest to begin with a quote from his second paper, (1924, page 62), which states his view on the difficulties in transition from a highly homogeneous amorphous solid, without even any scratches on its surface, to a more or less rough heterogen. (Griffith conducted his study on very carefully prepared glass specimens!) "The rupture strain energy of the strongest silica rods is so great that if it were all converted into heat it would make the rod red-hot. It is of interest to enquire, therefore, what becomes of this energy when fracture occurs. It is not converted into heat; indeed, measurements have shown that the defect of elasticity of the strong material is so inappreciably small that such an operation would take several minutes at least. What actually happens is that the energy is used up almost entirely in forming new surfaces, that is, surfaces of fracture, in the material. The disintegration of a strong drawn-down part is sometimes so complete that recognizable portions of this part are difficult to find. The sick ends also are usually broken, owing to the propagation of an elastic wave from the original fracture.

In concluding, I wish to refer to some of the difficulties which impede the further development of the theory of rupture. On attempting to pass from isotropic (amorphous) solids to brittle crystals we at once meet the difficulty that the surface tension is not a constant, but is a function of the position of the surface in the space lattice, so that the theoretical strength is different for different faces. The anisotropic nature of the elasticity is the further obstacle."

As we can see, Griffith saw very clearly the complexity and difficulties of transition from an isotropic material to a heterogen. It can also be said that these difficulties are not over even now, as shown by Mindess (1983).

1.3. *Tension under compression*

It is now clear enough that destruction in uniaxially compressed concrete is a result of *local transverse tension* [Berg (1950, 1971), Slate (1981, 1986), Delibes (1987)].

F. Slate *et al.* in (1986) drew the following conclusion:

"A tensile (or tensile-shear) mechanism is the most relevant crack mechanism controlling *failure of concrete in uniaxial compression*. This failure occurs in a direction perpendicular to applied load for all the concretes tested.

Normal strength concretes develop highly irregular failure surfaces including a large amount of bond failure. Medium strength concrete develop a similar mechanism, but at higher strain. The failure mode of high strength concretes is typical of nearly homogenical material. Failure occurs *suddenly* in a *vertical*, nearly *flat plane* passing *through the aggregate and the mortar*."

This fact is the basis of the approach, developed in Blechman (1988, 1989, 1992), where the nonlinearity of concrete behavior on the ascending branch of SSc is explained by microcracking.

Yet theoretically, in a continuous elastic solid under uniaxial compression transverse tensile stresses cannot be induced. An attempt to lay the responsibility on the existing oblique cracks is limited by the simple fact that they do not change, Slate (1981, 1986), under loading, but stay "dormant" up to stresses near the peak point of SSc. Instead of opening the initial microcracks, new microcracks appear, but they also are stable and do not grow, NMAB Report (1983).

The transverse tension also cannot be explained by this part of Poisson extension which is restricted by press-platens. As commonly known, *without restraints, Poisson extension does not induce stresses.* However, when the friction between platens and specimen is eliminated, the pattern of material splitting becomes especially clear. It should be noted that besides concrete and rock, other brittle materials, like ceramic and cast iron, fail in the same manner. If so, we should come to the conclusion that the mechanisms of failure of brittle solids are very general and independent from the individual features of concrete, rock and so on.

2. SPECIFICITY OF HETEROGENS

2.1. *General features*

- Heterogens are *isotropic in macro*, since their behaviour and strength are *independent* of the direction of loading.
- They are *heterogeneous and anisotropic in micro*, since they are built of *randomly oriented and randomly combined components*, whose properties are different when taken in the direction of the load.
- The internal order in artificial heterogens like ceramics and concrete is of a *heavily restricted stochastic* structure. Local parameters in it fluctuate only between given limits and mean parameters are kept in line with the technical requirements by rigorous quality control during the production process.
- *The intrinsic elastic modulus* of heterogen—E, measured under low-cyclic loading, *is constant*, as long as the integrity of the heterogen is retained, Karsan (1969).
- To study the brittle solids from nature (rock materials), we have to classify and sort them into *groups with the same structure*. Then the deviations of their features will be restricted, similar to artificial heterogens.
- Under short-term uniaxial load and normal temperature, a *heterogen has no plastic strains*, which can smooth out the influence of local gradients. Due to absence of plasticity, the tensile strength of brittle solids in macro is much lower than their compressive strength, in contrast to heterogen with high plasticity like soft metals, where the strengths in tension and compression are equal.

2.2. *Microcracking—fundamentals*

- The failure of a heterogen under compression is always preceded by microcracking, (see Berg (1950), Slate (1981a), Shah (1968b), Glucklich (1971b)).
- Microcracks *induced in a heterogen during loading* are local and stable. Their plane is parallel to the direction of maximal compressive stress. It has also been known for a long time, that under repeated load the microcracks are usually detected by acoustic emission, when the previously applied stress state is exceeded (Kaiser effect), Li and Nordlund (1993).
- *Microcracking is the reason for the nonlinearity* of the stress-strain curve at the ascending branch under short-term compression in both uniaxial and triaxial compression. It is clear now that there is no plasticity in the non-linear stage of loading, except for triaxial compression with high lateral stress.
- *Accumulation of dormant microcracks* gradually causes the heterogen to degenerate internally. This process is intrinsical and therefore is called "atrophy", not damage, which can be a result of external mechanical action. Failure sets in when the *limiting atrophy* is reached, which is the moment when the increment in loading energy absorbed by the heterogen equals the loss of energy due to its atrophy (degeneration), Blechman (1992).

2.3. *Integrity of heterogen*

The above mentioned features are effective up to the point of peak stress on the stress-strain curve. As is seen from the condition: $E = \text{const}$, integrity of a heterogen in the longitudinal direction is retained in the above domain, even when microcracks occur.

After the peak point of SSc the *macro* cracks usually split the heterogen into parts (blocks), which can still carry a decreasing load. The features of the macrocracked heterogen are essentially different and we have to treat it as a heterogen of a different kind. Therefore the models considered below are built for an integral heterogen existing in the domain before the peak point of the stress-strain curve. Due to this distinction in the features of the heterogen, one can hardly imagine a unified model valid for both—the ascending and descending branches of the SSc.

2.4. *Definition of heterogen*

For the proposed aim the following main features of the material, considered at the ascending branch of SSc, define a brittle heterogen: (a) it is isotropic in macro, (b) it is heterogeneous in micro, (c) its integrity is retained in the longitudinal direction of loading, (d) there are essential differences in Poisson's ratio and/or in the elastic moduli of its components, (e) its intrinsic elastic modulus is constant in macro, and (f) plasticity is absent under uniaxial compression.

2.5. *Origins and role of gradients*

The following main factors can induce gradient strains and stresses in a brittle solid under compression:

- Differences in Poisson's ratio and in the elastic moduli of the components.
- Residual stresses and local variations in density.
- Local tension around pores and flaws, Zaitsev (1981).
- Local shear due to gradients in shear modulus of the components.

Note: In this paper only the first item is treated, the residual stresses are taken under consideration in part 2.

When unconfined, Poisson extension (it is not tension!) in uniaxially compressed solids does not create stresses. However, in a restricted state gradients in Poisson extension induce transverse strains in accordance with Hooke's law. When this restriction is due to *differences in Poisson's ratio of the components*, their gradient will be expressed in very local combinations of compressive and tensile transverse strains. At the same time, differences in the elastic moduli of the components create heterogeneity in the strain-stress fields of the heterogen, which also induces lateral tension in it, as shown below.

3. LAYERED ELEMENT—"SANDWICH"

Let us begin with the phenomenon of gradient strains in a uniaxially compressed "sandwich", namely a multilayered element, built from two solids, indexed *a* and *m* ("aggregate" and "matrix") with Poisson's ratio $\nu_a > \nu_m$.

Cutting from the "sandwich" a part shown in Fig. 1, we can write the following equations: eqn (1) of continuity between the two layers, eqn (2) of equality in the increment of the gradient lateral forces—dF (transverse compression and tension) in these two layers and eqn (3) of the increment in the longitudinal strains, due to the isostress state of compression in the layers.

$$d\varepsilon_a^{ps} - d\varepsilon_a^* = d\varepsilon_m^{ps} + d\varepsilon_m^*, \tag{1}$$

$$dF_a = dF_m, \tag{2}$$

$$d\varepsilon_a = d\sigma/F_a, \tag{3a}$$

$$d\varepsilon_m = d\sigma/E_m, \tag{3b}$$

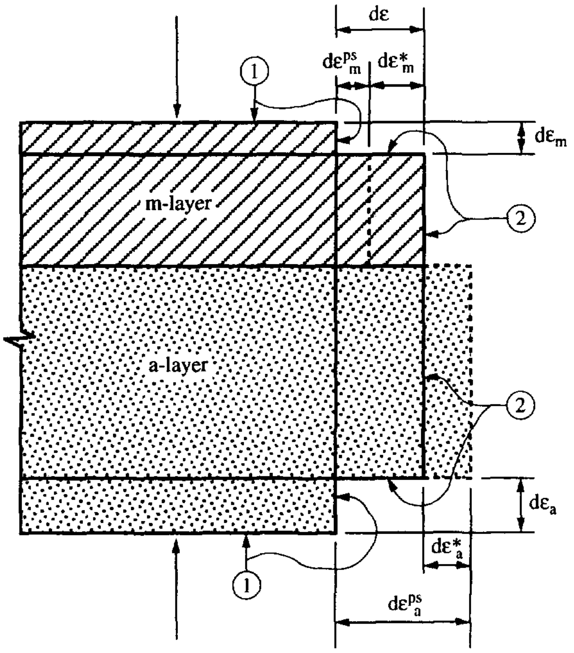

Fig. 1. Incremental strains in layered solid $d\varepsilon_m$, $d\varepsilon_a$—longitudinal strain of m- and a-layers; $d\varepsilon_m^{ps}$, $d\varepsilon_a^{ps}$—free Poisson extension of m- and a-layers; $d\varepsilon_m^*$—gradient strain of tension in m-layer; $d\varepsilon_a^*$—gradient strain of compression in a-layer; $d\varepsilon$—full extension. 1—Initial state, 2—compressed state.

where ε^{ps}—free Poisson extension and ε^*—transverse strain induced by gradient; $d\sigma$, $d\varepsilon$—the increments in longitudinal stress and strain, respectively.

The increment of the gradient forces can be expressed by the parameters of the layers, when their elastic moduli in the lateral direction are taken as equal to those in the longitudinal direction. (The layers are taken as isotropic in macro.)

$$dF_a = h_a E_a \, d\varepsilon_a^*, \tag{4a}$$

$$dF_m = h_m E_m \, d\varepsilon_m^*, \tag{4b}$$

where: E—modulus of elasticity, h_a, h_m—thicknesses of layers.

As follows from (2) and (4):

$$d\varepsilon_a^* = \frac{h_m E_m}{h_a E_a} d\varepsilon_m^*. \tag{5}$$

The factor:

$$\rho_o = \frac{h_m E_m}{h_a E_a}, \tag{6}$$

expresses the relationship between the stiffnesses of the two layers.

The increments in free Poisson extension of the layers are:

$$d\varepsilon_a^{ps} = \nu_a \, d\varepsilon_a, \tag{7a}$$

$$d\varepsilon_m^{ps} = \nu_m \, d\varepsilon_m. \tag{7b}$$

Substituting the above equations in (1) yields:

$$d\varepsilon_m^* = \frac{1}{1+\rho_o}\left(\frac{\nu_a}{E_a} - \frac{\nu_m}{E_m}\right) d\sigma. \tag{8}$$

The expression in brackets is a combi-gradient δ_s, which includes the influence of Poisson ratios and elastic moduli:

$$\delta_s = \frac{\nu_a}{E_a} - \frac{\nu_m}{E_m}. \tag{9a}$$

We will also denote:

$$\rho_m = \frac{1}{1+\rho_o}, \quad \text{and} \quad \rho_a = \frac{\rho_o}{1+\rho_o}.$$

Integrating (8) between the limits 0 and σ, with Poisson's ratio taken independent of σ, we obtain the gradient of transverse strain of tension, accumulated in the m layer:

$$\varepsilon_m^* = \rho_m \delta_s \sigma. \tag{10a}$$

The opposite gradient strain of transverse compression in the a-layer is

$$\varepsilon_a^* = \rho_a \sigma \delta_s. \tag{10b}$$

Equation (10a) can be also presented as

$$\varepsilon_m^* = \frac{\sigma}{E_a} \rho_m \left(\nu_a - \nu_m \frac{E_a}{E_m}\right). \tag{10c}$$

Then the expression in brackets is a Poisson gradient in the "sandwich", corrected in the second term by the factor E_a/E_b:

$$\delta_{s2} = \nu_a - \nu_m \frac{E_a}{E_m}. \tag{9b}$$

With $\varepsilon = \sigma/E_a$, the equation of gradient strain of local tension becomes

$$\varepsilon_m^* = \rho_m \delta_{s2} \varepsilon. \tag{10c}$$

In the compressed layer the gradient strain is:

$$\varepsilon_a^* = \rho_a \varepsilon \delta_{s2} = \rho_o \varepsilon_m^*. \tag{10d}$$

As can be seen from eqn (9) with some proportionality between two pairs of Poisson ratio and elastic moduli, the gradient δ_s in the "sandwich" can be very slight. For example: if $\nu_a = 0.26$, $\nu_m = 0.13$, when $E_a = 40{,}000$ MPa, $E_m = 20{,}000$ then $\delta_s = 0$!

When the "sandwich" is built from crystalline brittle solids, it is quite possible that the gradient factors at crystals level—δ_{acr} (described below in Section 5), will be higher than between the layers. Then the "sandwich" will be split due to intercrystalline gradients described below, not due to interlayer differences.

4. COMPRESSED MATROGEN

4.1. *Poisson's gradient in matrogen*

In matrogens, unlike the "sandwich", the aggregate particles are surrounded by a matrix and therefore their longitudinal strains, not the stresses, are almost equal. Then, instead of eqn (3) the increments $d\varepsilon_a$, $d\varepsilon_m$ in their longitudinal strains have to be written as:

$$\frac{1}{k_a}d\varepsilon_a = \frac{1}{k_m}d\varepsilon_m = d\varepsilon, \tag{11}$$

where k_a, k_m are a deviation of the components from the average increment in the material.

In an artificial matrogen, like concrete, the granulometry, i.e., the composition of aggregate of different size, is kept under very close control. At the same time the dispositions of the large aggregate particles is random enough for assuming the same average value for the stiffness ratio of the components—ρ_o at every cross-section, as follows

$$\rho_o = \frac{k_m v_m^{2/3} E_m}{k_a v_a^{2/3} E_a}, \quad \text{when } v_a + v_m = 1. \tag{12}$$

Here, v_m and v_a are volume fractions of matrix and aggregate, respectively. The matrix comprises the hardened cement paste, sand and voids. For a matrogen, eqn (2) remains in effect and yields:

$$d\varepsilon_a^* = \rho_o \, d\varepsilon_m^*. \tag{13}$$

The free Poisson extension of the components will be:

$$d\varepsilon_m^{ps} = \nu_m \, d\varepsilon_m$$

$$d\varepsilon_a^{ps} = \nu_a \, d\varepsilon_a$$

where ν_a, ν_m are the Poisson ratios of the aggregate and matrix, respectively. Equation (1) is also in effect for a matrogen, so substituting the found expressions in (1) we have

$$d\varepsilon_m^* = \frac{k_a \nu_a - k_m \nu_m}{1+\rho_o} d\varepsilon. \tag{14}$$

Taking $\rho_m = 1/(1+\rho_o)$ and integrating (14) from $\varepsilon = 0$ to ε we obtain the equation for gradient strain of local transverse tension—ε_m^* induced in the matrix:

$$\varepsilon_m^* = \rho_m (k_a \nu_a - k_m v_m)\varepsilon. \tag{15a}$$

The expression

$$\delta_\nu = (k_a \nu_a - k_m \nu_m)\rho_m, \tag{16}$$

can be defined as the *gradient factor in matrogen*. Then the gradient tensile strain is

$$\varepsilon_m^* = \delta_\nu \varepsilon. \tag{15b}$$

Then the gradient of the compressive strain will be

$$\varepsilon_a^* = \rho_o \varepsilon_m^* = \rho_o \, \delta_\nu \varepsilon. \tag{15c}$$

Let us attempt a rough estimation of the gradient factor in concrete, with $\nu_m = 0.14$ and

$\nu_a = 0.24$, Neville (1971). For $E_a = 40{,}000$ MPa, $E_m = 15{,}000$ MPa, $v_a = v_m$, $k_a = 1.05$, $k_m = 0.95$ we have $\rho_0 = 0.375$ and $\rho_m = 0.73$. Then the gradient factor will be: $\delta_v = 0.73*(0.24*0.95-0.14*1.05) = 0.06$.

4.2. *Poisson gradient—alternative approach*

In considering the gradient strain, we can try another approach, based on the apparent value of Poisson's ratio of the matrogen—ν_o, measured in tests. Then the average gradient strain—ε^* induced by Poisson's ratio between the matrix and aggregate can be estimated as

$$\varepsilon_m^* = (\nu_o - \nu_m)\varepsilon. \tag{17}$$

The condition of continuity is "built in" in (17), as the apparent value of Poisson's ratio was taken. Under this approach the gradient factor is defined and calculated as

$$\delta_v = \nu_o - \nu_m. \tag{18}$$

Since Poisson's ratio for concrete is about 0.20 and, as given above, $\nu_m = 0.14$, the second estimation yields similar result to the above $\delta_v = 0.20-0.14 = 0.06$.

4.3. *Significance*

The importance of the Poisson gradient factor lies, at first, in the possibility of explaining the process of microcracking and predicting the critical loading strains. However, due to stochasticity of the microcracking process, the gradient models cannot be simply used to explain the behavior and strength of concrete. In part 2 of this paper the solution of this problem is given.

On the other hand models can be used for revealing the resistance of the heterogen to microrupture. For example, if $\varepsilon_a = 1.7*10^{-3}$ and $\delta = 0.06$ then the critical lateral gradient strain, which will induce microrupture, can be predicted as $\varepsilon_{crit}^* = 1*10^{-4}$, which falls within the well known limits 0.5–$1.5*10^{-4}$ millistrain for concrete tensile strain at failure. Or, when we know the critical strain of microcrupture in a heterogen, we can predict the limit of linearity in the SSc.

4.4. *Thrust in matrogen*

When the elastic moduli of matrix and aggregate in a matrogen are widely different (as in low-strength concrete), heterogeneity of the stress field will create local domains of thrust between aggregate particles.

To model the stress gradients induced by thrust in a uniaxially compressed heterogen we will consider the large aggregate particles as spheres of radius—r, arranged as shown in Fig. 2 in layers at distance H in the vertical and L in the horizontal direction, with the particles in a pyramid pattern. Here $L > 2H$ and between every pair of layers of aggregate there is a cushion-layer of matrix. The heterogen is under a longitudinal compressive strain ε.

When the elastic modulus of the matrix is less than that of the aggregate, the stresses in the aggregate are higher than that in the matrix. Taking the stresses in matrix as references we can find the excess compressive stresses in the large particles for a unit cross-section of the element from the following expression:

$$\Delta\sigma_c = k_1 v_a^{2/3}\varepsilon(k_a E_a - k_m E_m), \tag{19}$$

where k_1 is a factor of the disorder for the large particles and of dissipation of the excess stress over the fine-size particles.

Then, the thrust in the base of a pyramid for this scheme will be

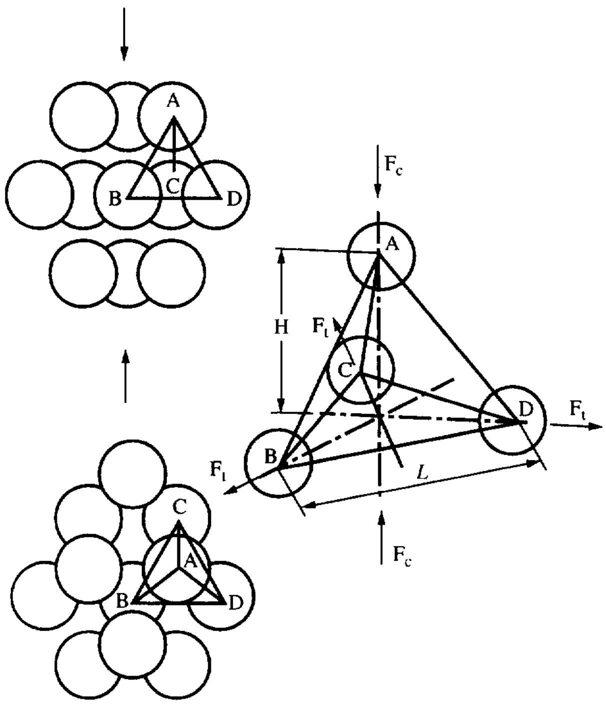

Fig. 2. Pyramid scheme of aggregate particles and thrust in matrogen.

$$\sigma_t = k_t \Delta\sigma_c, \tag{20a}$$

or

$$\sigma_t = k_t k_1 k_m v_a^{2/3} \varepsilon \left(\frac{k_a}{k_m} E_a - E_m \right). \tag{20b}$$

Here, k_t is also a factor of disorder, but now related to the geometry of the pyramids. Denoting $k_a/k_m = k$, and taking $K_{tr} = k_t k_1 k_m$, we obtain

$$\sigma_t = K_{tr} v_a^{2/3} \varepsilon (k E_a - E_m). \tag{20c}$$

In strain terms, dividing (20c) by E_m, we have

$$\varepsilon_t = K_{tr} v^{2/3} \varepsilon \delta_E, \tag{21a}$$

where the gradient factor of thrust δ_E is

$$\delta_E = k \frac{E_a}{E_m} - 1. \tag{21b}$$

In the bulk of the heterogen the thrust in every "pyramid" is neutralized by the opposite thrust of the "neighbors". The thrust can manifest itself only at the free edges of the specimen and due to space-disorder of aggregate particles.

Estimation of K_{tr} *for concrete*. To estimate the K_{tr} we will use the state of local rupture where

$$\varepsilon^R = K_{tr} v^{2/3} \varepsilon_p \delta_E, \quad (22)$$

ε^R being the limiting strain of microrupture (taken as $1.0 * 10^{-4}$). The peak strain in concrete under compression—ε_p, is $2.2 * 10^{-3}$ and again $k = 1$, $E_a = 40{,}000$ MPa, $E_m = 15{,}000$ MPa, hence $\delta_E = 1.667$ and $K_{tr} = 0.03$.

A possible explanation for this very low value of K_{tr}, is that most of the thrust is dissipated over the fine grains and due to the disorder in the arrangement of large particles.

5. CRYSTALON UNDER COMPRESSION

5.1. *Acrons*

As defined above, a crystalon is a heterogen built from randomly oriented crystals. Usually the Poisson's ratios related to the main axes of a single crystal are very distinct. Therefore, in crystalon under compression, gradient strains appear and two populations of *laterally tensioned* and *laterally compressed* crystals are created. As the origin of the crystalon's atrophy, the laterally tensioned population is of highest importance. Being an antithesis to the laterally compressed particles they can be called *a*nti-*c*rystals + *on* = *acrons*, the suffix "on" being a common part of the particle nomenclature. Non-symmetric gradients, which can induce other than compression-tension states in crystals, are not considered here.

5.2. *Elastic gradients in acrons*

In contrast with the matrogen whose particles and matrix are homogeneous in macro, irrespective of the direction of the load, the acron strains in a crystalon depend on its angle with the direction of the main stress.

To estimate the gradients induced between the acrons and their environment due to the difference in elastic moduli and Poisson's ratio in the elastic stage, a simple model of a symmetrical acron is taken, as shown in Fig. 3. Its main axes a and b are at the angle α to the direction of the main compressive strain ε_1. The axis c is horizontal and at the right angles to a and b and to the axis "1". The elastic moduli and Poisson ratios of the acros are: E_a, v_a, E_b, v_b, E_c, v_c, when $E_c = E_b$, and $v_c = v_b$.

The shear stresses and shear-induced gradients (due to differences in elastic moduli along the axes 1, 2, 3 of the acron) are not taken here into consideration since these gradients are zero for $\alpha = 0$, when the Poisson gradients are maximal. Shear gradients have their maximum at $\alpha = 45$ deg, but then the Poisson gradients are minimal and the contribution of shear gradients is negligible.

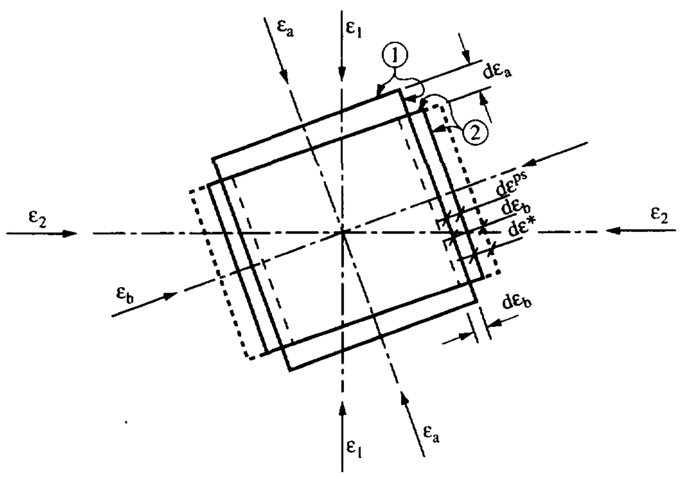

Fig. 3. Incremental gradient strains in acron $d\varepsilon_a$, $d\varepsilon_b$—infinitesimal longitudinal strains in 'a' and 'b' directions; $d\varepsilon^{ps}$—free Poisson extension of acron; $d\varepsilon$—effective extension of acron; $d\varepsilon^*$—gradient strain of tension in acron.

The bulk of the crystalon around the acron is taken in whole with its average parameters E_o, ν_o in all directions. By definition of the acron $\nu_a < \nu_o < \nu_b$. The element is under longitudinal strain ε_1 and lateral strains $\varepsilon_2 = \varepsilon_3$, with

$$\varepsilon_2 = \omega\varepsilon_1. \tag{23a}$$

According to Fig. 3 the infinitesimal strains for axes a and b are:

$$d\varepsilon_a = (\cos^2\alpha + \omega\sin^2\alpha)\, d\varepsilon_1. \tag{23b}$$

$$d\varepsilon_b = (\sin^2\alpha + \omega\cos^2\alpha)\, d\varepsilon_1 \tag{23c}$$

and

$$d\varepsilon_c = d\varepsilon_3. \tag{23d}$$

Denoting

$$f_a = \cos^2\alpha + \omega\sin^2\alpha.$$

$$f_b = \sin^2\alpha + \omega\cos^2\alpha.$$

We have

$$d\varepsilon_a = f_a\, d\varepsilon_1. \tag{23e}$$

$$d\varepsilon_b = f_b\, d\varepsilon_1. \tag{23f}$$

$$d\varepsilon_c = \omega\, d\varepsilon_1. \tag{23g}$$

The lateral strain—$d\varepsilon_a$—for a free single crystal is:

$$d\varepsilon_a = \nu_a\, d\varepsilon_a - d\varepsilon_b + \nu_c\, d\varepsilon_c. \tag{24a}$$

Replacing the single crystal by the bulk material we can find the lateral deformation of this "bulk" crystal $d\varepsilon_o$ as

$$d\varepsilon_o = \nu_o\, d\varepsilon_a - d\varepsilon_b + \nu_o\, d\varepsilon_c. \tag{24b}$$

The strain gradient of tension between the bulk and the single crystal—$d\varepsilon^*$—is

$$d\varepsilon^* = d\varepsilon_o - d\varepsilon_a. \tag{25a}$$

It should be noted that the condition of continuity is preferred in these equations, because the actual values of ν_o and E_o in eqns (24b) express the real interaction of the crystalon's components. Substituting eqns (24) in (25a) we obtain

$$d\varepsilon^* = (\nu_o - \nu_a)\, d\varepsilon_a - (\nu_c - \nu_o)\, d\varepsilon_c. \tag{25b}$$

The above differences can be denoted as

$$\delta_a = \nu_o - \nu_a,$$

$$\delta_c = \nu_c - \nu_o.$$

Using the above definitions and the condition (23g) we can rewrite (25b) as

$$d\varepsilon^* = (f_a\,\delta_a - \omega\,\delta_c)\,d\varepsilon_1. \tag{25c}$$

The expression in brackets is the acron's *gradient factor*—δ_{acr}

$$\delta_{acr} = f_a\,\delta_a - \omega\,\delta_c. \tag{26}$$

and then (25c) can be rewritten as

$$d\varepsilon^* = \delta_{acr}\,d\varepsilon_1. \tag{27}$$

In (27) δ_{acr} is independent from ε_1. Integrating (27) in the limits $\{0\text{–}\varepsilon_1\}$ we will find the local lateral gradient strain as the following

$$\varepsilon^* = \delta_{acr}\varepsilon_1. \tag{28}$$

It should be noted that by definition of the acron, the gradient δ_{acr} is positive

$$\delta_a f_a - \omega\,\delta_c > 0.$$

This gradient strain between the acron and the bulk is maximal when $\sigma_2 = 0$, $\alpha = 0$. Then its value will be

$$\varepsilon^* = \varepsilon_1(v_o - v_a), \tag{29}$$

which corresponds to eqn (17) for a matrogen.

The submicrocracks found by Attiogbe and Darwin (1987) seems to be induced by the acron-kind mechanisms.

5.3. *Releasable energy in acrons*

5.3.1. *Influence of geometry*. In seeking the limiting state for heterogen microrupture, we have to note that in contrast with Griffith's approach, which considers an infinite half-space of tensioned isotropic material with one or more small initial cracks, here we consider a longitudinally compressed crystalon without initial cracks, namely a population of small *laterally tensioned* crystals—acrons, tightly glued in *laterally compressed* surroundings.

Two schemes are basic here: one when the contact zone around the acron is weaker than the acron itself and the other when the acron is weaker than its surroundings. Since the microcracking is release of the tensile energy accumulated in the acron, the condition of limiting strain should be supplemented by the energy balance in it. Due to the rigid confinement of the acron, only part of this energy can be released in a microcrack.

For a rough, preliminary estimation of the link between the geometry of the acron and the energy available for microcracking we will consider a single acron as a prism with dimensions: a, b, h, when $a > b$. The boundary plane between the zone of releasable energy and the confined zone in acron is taken at 45 grad. The x, y axes are chosen from the condition $v_y > v_x$, so that the strain energy is released along axis x. It then suffices to check four possibilities: $a \parallel x$ and $b \parallel x$, for $h \geqslant b$ and $h < b$.

The results show that the volume of the domains of releasable strain energy in acrons can vary widely from 5 to 42%, depending on geometry and orientation of the acron, and influence seriously the strength of heterogen under compression.

5.3.2. *Lateral strain of rupture*. The releasable energy of tension accumulated in an acron by the lateral gradient strains, ε^*, (per unit side area) is

$$\mathscr{E}_{acr} = (\varepsilon^*)^2 E_{acr} t_{acr}, \tag{30a}$$

where E_{acr} is the elastic modulus of the acron and t_{acr} is the average width of the releaseable-energy zone.

At the critical state of microrupture, the gradient strain has to equal that of resistance of the contact layer to tension, ε_{cont}. The energy $\mathscr{E}_{cont}$ needed to rupture the contact layer will be

$$\mathscr{E}_{cont} = (\varepsilon^R_{cont})^2 E_{cont} t_{cont}, \tag{30b}$$

where E_{cont} is the elastic modulus of the contact layer and t_{cont} its width.

Rupture takes place when the energies are equal

$$\mathscr{E}_{cont} = \mathscr{E}_{acr}.$$

Then the critical strain of rupture, ε^*_{crit} will be

$$\varepsilon^*_{crit} = \varepsilon_{cont} \sqrt{\frac{t_{cont}}{t_{acr}} \frac{E_{acr}}{E_{cont}}}. \tag{31a}$$

Under the second scheme (weak acron), the critical strain is the limiting strain of rupture of the acron itself

$$\varepsilon^*_{crit} = \varepsilon^R_{acr}. \tag{31b}$$

5.4. *Multi-acrons*

Multi-acrons can be defined as a chain or tree of contacting acrons with the same E and v, but of different slope to the direction of σ_1. Naturally, in the chain the ratio $h/\Sigma b$ (Σb being the length of the chain) will be very low and, in this case, as shown above, the accumulated tensile energy which can be transmitted through this chain will be, at most, the releasable energy of a single crystal.

Moreover, with its random structure, a multiacron will have a broken, dentale shape, where every acron is anchored by its laterally compressed neighbors, actually unable to transmit the gradient strain to the next acron in the chain. Thus, in a multiacron, the possible point of microcracking is a contact between two acrons, especially if we bare in mind the doubled releaseable tensile energy, at that point. As a result, we can consider that existence of multiacrons can give rise to the process of micocracking at earlier stages of the loading and reduce the strength of the crystalon.

When a crystalon contains several kinds of dispersed crystals with different v and E, we have to superimpose the probability density functions (pdf) of the strain gradients induced by each of these kinds.

5.5. *Poisson pistons*

Olaffson and Peng (1976) described some microcracking mechanisms in a mono-crystalline solid—Tennessee marble, tested in uniaxial and also in triaxial compression with high lateral pressure (Fig. 4). They found that the induced microcracks (not the lamellae) under all kinds of loading were oriented close to the direction of maximum compression, demonstrating the action of local lateral tension. Since the nonlinear stage of degeneration (atrophy) of a crystalon determines its bearing capacity (see part 2), it is important to check the microcracks not at the peak point or even in the after-peak stage, but first of all during the nonlinear stage of the ascending branch of the stress-strain curve. In the center of picture A of figure 6 of their paper, a clear vertical microcrack splits a grain at this stage (as can be seen from the graph in this figure), its location apparently determined by twin lamellae from the contacting grains. These cracks were frequently observed and their nucleation mechanism was called type I.

The second frequently observed mechanism is of laterally expanding grains, which clearly ruptured their surrounds by vertical microcracks. They act as pistons, and so we will call them. At the elastic stage single crystals will behave as pistons, when their axis of maximum Poisson ratio v_p is nearly laterally oriented. In this case the gradient factor, which can be called the *piston gradient*, can be approximated as

$$\delta_{pst} = v_p - v_o. \tag{32a}$$

For example, for $v_p = 0.3$ and $v_o = 0.2$ the $\delta_{pst} = 0.10$!

Mechanism IV is presented by Olaffson and Peng as a case of plasticity, when twin gliding creates lamellae, Poisson's ratio of the grain increases up to 0.5, and then we have a *piston gradient of plasticity*, δ_{pp}, which is especially high:

$$\delta_{pp} = 0.5 - v_o. \tag{32b}$$

For $v_o = 0.2$ we have $\delta_{pp} = 0.30$(!!). This very large gradient can explain the strong piston effect in the lateral direction, which induces not one, but a number of cracks in the neighboring grains, as the pictures in the above paper show. The question is whether this takes place within the ascending nonlinear part of SSc or at the after-peak stage (descending branch), where it is no longer relevant in the bearing capacity context in its usual meaning. In any case, the population of "pistons" can add a good part to the process of microcracking and degeneration (atrophy) of a crystalon.

6. MICROCRACKS

6.1. *Opening*

In contact-type rupture one zone of an acron's releasable energy is active. By contrast, when the body of an acron or a multiacron is ruptured, the tensioned releasable zones are doubled and the crack opening can be doubled compared with the contact-type case. It is of interest to estimate the microcracks' opening. In a matrogen (like concrete) the length of the microcracks is of 3–6 mm, Slate *et al.* (1981a,b). They are usually found on the

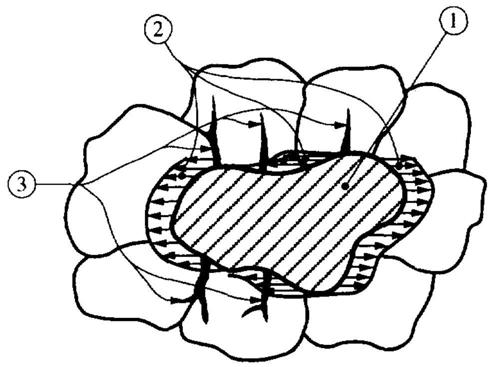

Fig. 4. Scheme of "piston-grain" action (1) piston-grain (2) distribution of piston pressure (3) microcracks in surrounding grains.

interface between aggregate and matrix and therefore are of the contact rupture type. Taking the depth of the releasable zone as half their length (45 grad!) and the limiting strain of microrupture $\varepsilon_{tp} = (1–2) * 10^{-4}$, we find that the opening is $0.15–0.6 * 10^{-3}$.

In a crystalon with cubic crystals of about 1*1*1 mm, in contact-type rupture, when $\varepsilon_{tp} = (1–2) * 10^{-4}$ the opening will be about $(0.5–1.0) * 10^{-4}$, and in a internal rupture of a multiacron it will be $(1–2) * 10^{-4}$. Attiogbe and Darwin (1987) investigated submicrocracks in hardened cement paste and found their opening in the above order.

6.2. *Locality and stability*

The situation where acrons in a compressed crystallon are isolated by laterally compressed surroundings explains the phenomenon of locality and stability of microcracks. The compressed surroundings restrict and even arrest the development of a microcrack at the moment of its appearance. Microrupture of an acron (or of its contact) eliminate not only the releasable tension induced by gradients, but even the possibility of the gradients to appear anew at the ends of the microcrack in spite of continuous Poisson extension during the loading. At the same time other acrons with a lower gradient factor reach the critical strain of local microrupture and after appearance of microcracks they also sink into the stable state and remain dormant almost up to the peak stage while the limiting atrophy is reached.

6.3. *Merging*

In the atrophy approach, the *bearing capacity* of heterogen is exhausted when, due to progress in microcracking process, it reaches its atrophy limit, as described in Blechman (1992). The distribution of microcracks according to type and its change with increase in stresses, for different stress states and for concrete of different strength and types were thoroughly investigated by Slate (1981a). He had found the process of "bridging", coupling of stable microcracks in the last step of microcracking process, without their merging into macrocrack. It means that we can work on the problem of bearing capacity without treating the microcracks interaction. In other words, when searching after the *bearing capacity of brittle solid only*, we do not need to know where and how the main macrocrack will split the specimen (from the data we know it), because it takes place at the very end and even after the process of exhaustion of the bearing capacity of the material.

7. CONTRASTS

The conditions, under which the micromechanical mechanisms operate, are induced by combination of the characteristics of heterogen's components. Moreover there is a "competition" among the mechanisms. The one which induces the largest gradients "wins" the competition and will be (usually alone) the origin of microcracking.

Crystalon vs matrogen. In a crystalon under compression the Poisson mechanism and the pistons are the only origin of heterogen's degeneration, when the laterally tensioned particles, acrons or multiacrons, are surrounded by a mass of laterally compressed crystals and especially when they are overlapped by pistons. By contrast, plain concrete is a matrogen, where the particles of aggregate, with Poisson's ratio higher than that of the matrix, are compressed and the matrix around them tensioned. In concrete with lightweight aggregate, where Poisson's ratio of the strong matrix is higher than that of the aggregate, the latter will be tensioned and matrix compressed.

Comparing the gradients we can see that in a matrogen of the kind of low-strength plain concrete, with large difference in elastic moduli between matrix and aggregate, and *relatively low adhesion between them*, the thrust mechanism is critical and will destroy the material. Even in this state participation of the Poisson's mechanism has to be taken into consideration.

According to our observations, when the friction between concrete specimen and the platens of the testing set-up is eliminated, the tested cubes are split by vertical cracks into four to six parts. It happens with concrete of strength from 20 MPa upwards and is especially noticeable in high-strength concrete of 60–100 MPa. Here the Poisson mechanism dominates strongly in the atrophy process and the splitting phenomenon is pronounced, Slate (1981, 1986), showing clearly the transition from thrust-type to crystalon-type rupture.

Micro vs macro. The parameters in the above *gradient micromodels* are measurable and can be found from tests. The critical problem of their statistical character and link with the *macroparameters* of atrophy is the subject of part 2 of this paper.

Shear vs tension. As one can see, the limiting shear stresses do not appear in the gradient models of microcracking. It corresponds to the findings of Blechman (1997), which show that the resistance of a brittle solid to shear stress is very high and the stage of degeneration due to microcracking is reached first. Only when microcracking loosens the heterogen can the shear forces go into action.

Dispersed tension. When $v_a > v_m$, the local tension induced by thrust between lateral sides of the particles in a matrogen is combined with the tension induced above and below the particles due to gradient in Poisson's ratio. These two kinds of gradient tension vary differently during the stage of microcracking. Cracking of the spots tensioned by the Poisson gradient, eliminates the tension in it, but only a small part of these spots are cracked. Where the microcracks cut the thrust spots their mutual neutralization is weakened, and then the thrust-induced transverse tension is dispersed laterally through the uniaxially compressed heterogen.

During the loading, the lateral strains gradient and microtensions arise and local lateral tension increases. As a result, the particles in the matrogen are surrounded by a matrix compressed in the longitudinal direction, but is under dispersed tension in lateral direction, which overlaps the local gradient strain.

As is well known, the correct pattern of failure of a brittle solid under compression after accumulation of microcracks, is its splitting by one or more vertical macrocracks, which occur at the peak point of the load. In the traditional approach to this phenomenon the dispersed lateral macrotension in the matrogen can be the missing factor, but in crystalon-type rupture the splitting cannot be explained in this way.

8. SUMMARY

A review of existing models of the behavior and strength of brittle heterogeneous solid (heterogen) shows their weak physical base. When built in general they do not fit the data well and do not explain the appearance of microcracks without initial cracks and pores.

Griffith's approach, which assumes a tensioned, homogeneous half-space with initial cracks perpendicular to the direction of main stress is a case in point. In the present work, instead of abstracting the brittle solid as a homogeneous material, it is assumed to be heterogeneous from the beginning, together with the paradigm that its failure under compression is always preceded by appearance of microcracks. These microcracks are local, stable (dormant), and uniformly distributed, their plane is parallel to the direction of maximal compressive stress. The intrinsic elastic modulus of heterogen is constant up to the peak load.

It is shown that gradients in Poisson's ratio and in the elastic moduli of the components of the brittle solid can explain the phenomenon of its microcracking in compression. A number of gradient mechanisms were modelled here: the Poisson mechanism in layered material ("sandwich"); the Poisson and thrust mechanisms in a matrogen, for example as concrete; the Poisson gradients of tension in a crystalon, which create "acrons"—laterally tensioned crystals, because of *minimal* Poisson's ratio in their lateral direction, when their surroundings are laterally compressed, and also the mechanisms of "pistons", (crystals with *maximum* Poisson's ratio in the lateral direction), which can go to the state of plasticity and crack the neighboring crystals.

The conclusions which can be drawn from these mechanisms are:

• The gradient models can explain the appearance of microcracks and their features (especially their locality and stability) in a compressed brittle solid, based on its known characteristics.

• These models, and especially the mechanisms of Poisson's gradient, are universal. They affect every heterogen under compression: rock materials, concrete, ceramics etc.

• The gradient mechanism does not need initial cracks for inducing and realizing the process of microcracking and degeneration.

• Gradient mechanisms are stochastic; they do not cause *macrocracks*, but induce a lot of stable *microcracks*, in good accordance with a vast number of experiments.

• The gradient models are descriptive, based on clear mechanics and on the measurable parameters of the heterogen.

• The thrust mechanism is important in matrogens (like low strength concrete), when a large difference in the elastic moduli of the components exists.

• In a compressed crystalon (brittle solid built from randomly oriented crystals), a population of *laterally tensioned crystals*, called "acrons", is created. The models of gradient strain in the acrons are given, including the equation of critical strains. Since the acrons are glued in a fully compressed environment, the question of releasable energy in the critical state was checked.

• In contrast to acrons, the "pistons", laterally overcompressed crystals or grains, due to high positive gradient with laterally compressed surroundings, can crack their overlying and underlying neighbors. The action of pistons is especially strong where, due to transition into the state of plasticity, their Poisson's ratio rises up to 0.5.

• Gradient mechanisms suffice to cause degeneration in a heterogen and exhaust its bearing capacity under increasing compression, without recourse to shear stresses.

• The gradient models show that the strength of crystalon increase when the crystals are of minimal dimensions and have minimal differences in the mechanical characteristics along their main axes and when their limiting strain of sliding (plasticity) is as close to the maximal value as possible for given atomic composition.

• Corroboration of the role of gradient-strain mechanisms in the behavior of a heterogen under load can be seen in the success of this approach in solving of two theoretically interesting and practically important old problems: one of bearing capacity of brittle solids and granular materials in triaxial compression, part 1 in Blechman (1997); and the second of modelling the lateral stress release, part 2 in Blechman (1997). The solution of the latter shows that the earth's crust is always in the limiting state of its bearing capacity and explains, in good accordance with well known facts, that even a small drop in lateral compression in the crust suffices to induce an earthquake.

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